De-Stress by Coloring

Leader Checklist

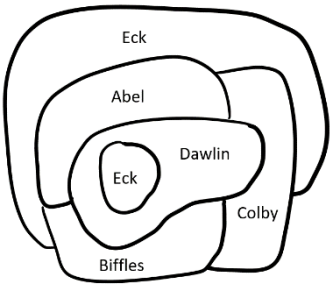
- ☐ Read through the module.
- ☐ Make copies of the coloring sheets at the end of the module.
- ☐ Gather additional supplies: Crayons, colored pencils, or markers

- Choose a coloring sheet[1] and dive in! It's a very calming way to pass the time.

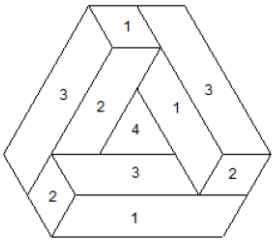
- Challenge #1: Make sure that no two adjacent regions have the same color. That is, if two regions share a boundary, they should be different colors. (If they just meet at a corner, that's okay – they can be the same color if you like.)

- Challenge #2: Carry out Challenge #1 while using the smallest possible number of colors.

- A map maker might be very interested in these challenges!
  - If you're coloring countries on a map, it's important to make sure that neighboring countries are different colors. In general, how many colors do you need in order to do this?
  - What about this map? How many colors do you need?
  - Notice that the country of Eck is in two separate pieces, just like the state of Michigan.
  - That's a rare situation – let's rule that out. On maps where every country is a single connected region, how many colors do you need? Draw some make-believe maps and try to color them using the minimal number of colors. Can you always do it in three colors? Four? Five?

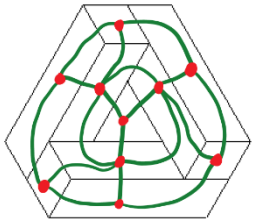


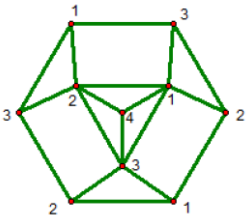
- If you're getting tired of coloring, you can switch to using numbers, like in the image to the right.



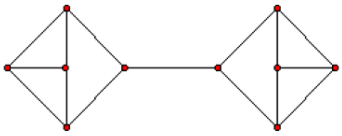
[1] All attached coloring sheet images are taken from the website https://clipart-library.com/optical-illusion-coloring-pages-printable.html .

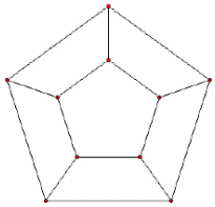
- If you think about it, the actual shapes of the countries aren't important. What matters is which countries are next to each other.

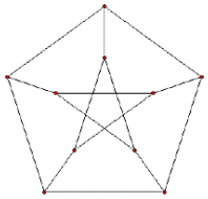




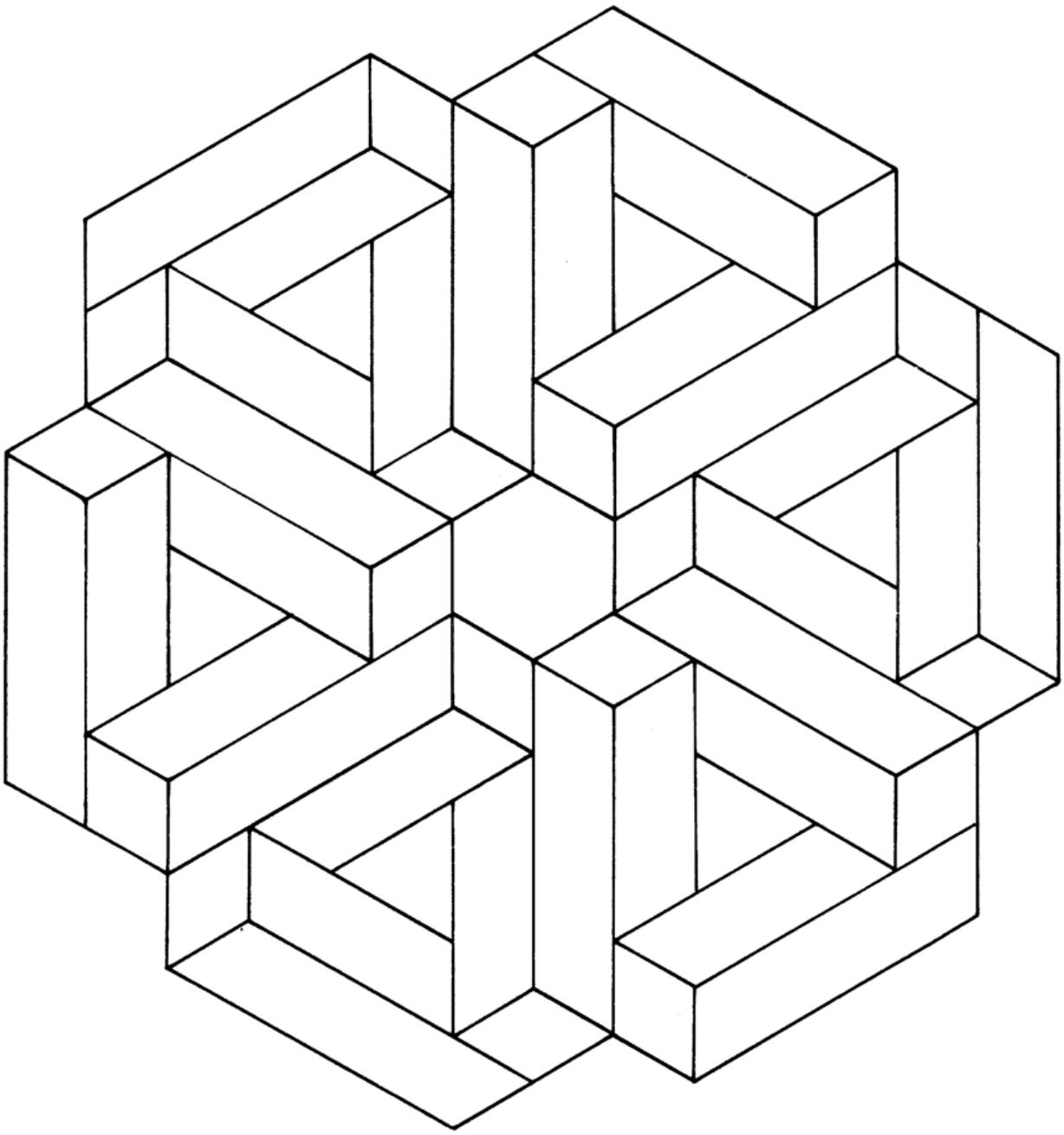
  - We can represent each map with a *graph* (a collection of *vertices*, some of which are joined by *edges*). Each country is represented by a vertex, and two vertices are joined by an edge exactly when the two countries share a border.
  - Coloring the countries on the map corresponds to labeling the vertices in the graph in such a way that no two adjacent vertices have the same label. Such a labeling is called a *proper vertex labeling* of the graph.
  - What's the smallest number of labels we need to do this? That's the *vertex chromatic number* of the graph. Can you find the vertex chromatic number of each of these graphs?



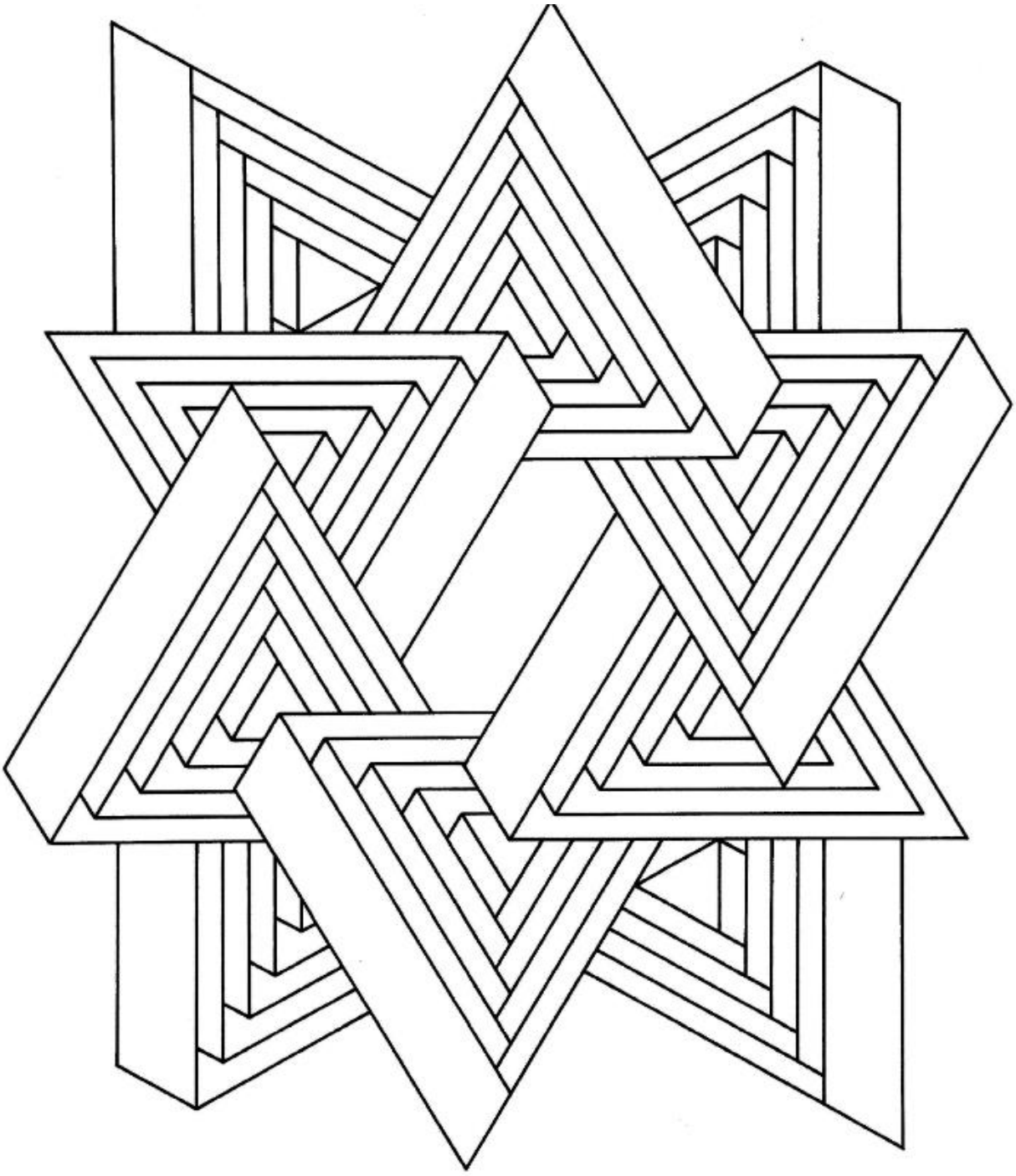




- Follow-up:
  - Read about the Four Color Theorem and its history. Who first claimed to have proved it? Who finally did prove it?
  - You can also label the *edges* of a graph so that no two adjacent edges have the same label. Such a labeling is called a *proper edge labeling*, and the *edge chromatic number* of the graph is the smallest number of labels that we need to do this. Can you find the edge chromatic numbers of the graphs above?



https://clipart-library.com/optical-illusion-coloring-pages-printable.html



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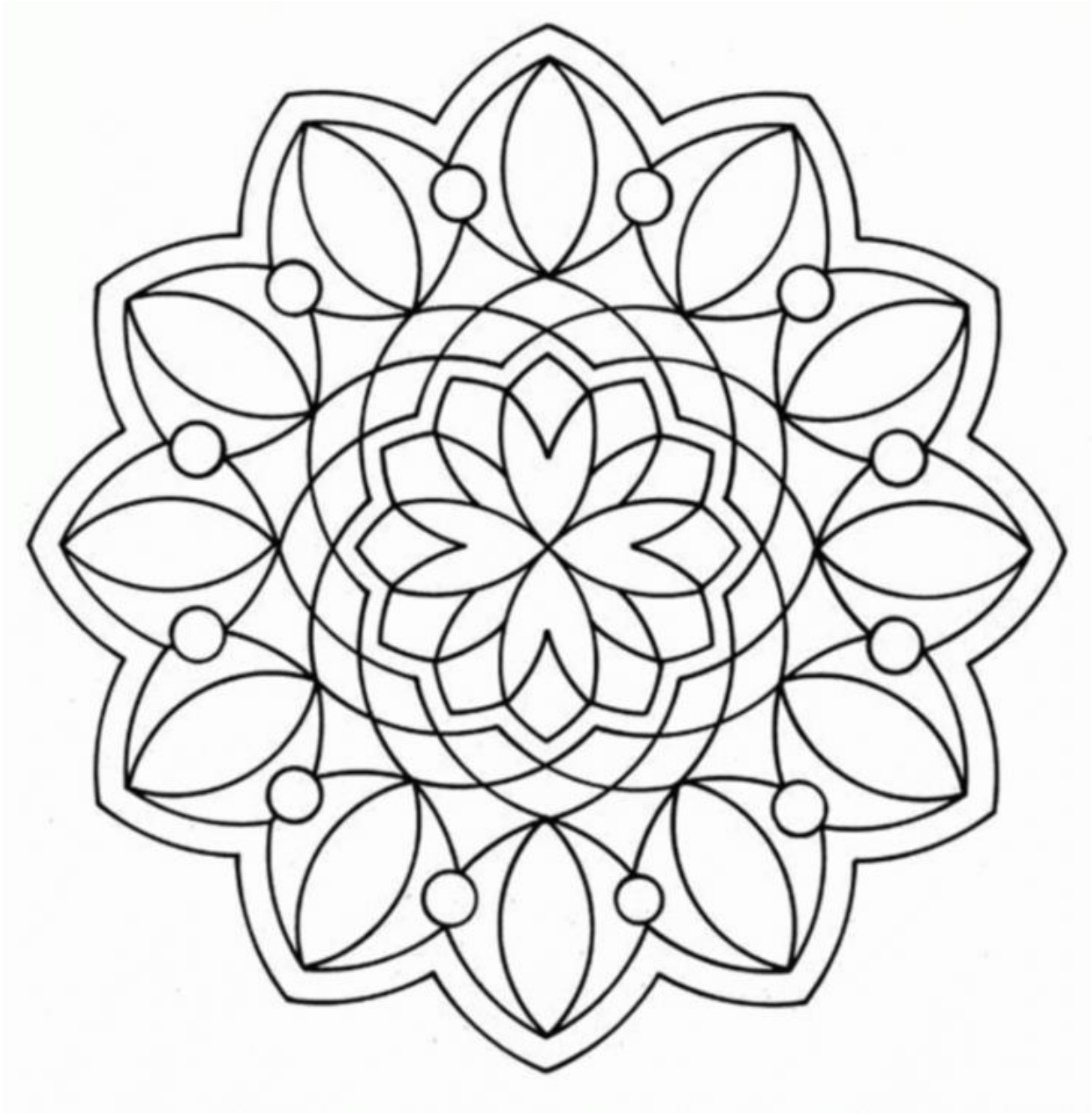

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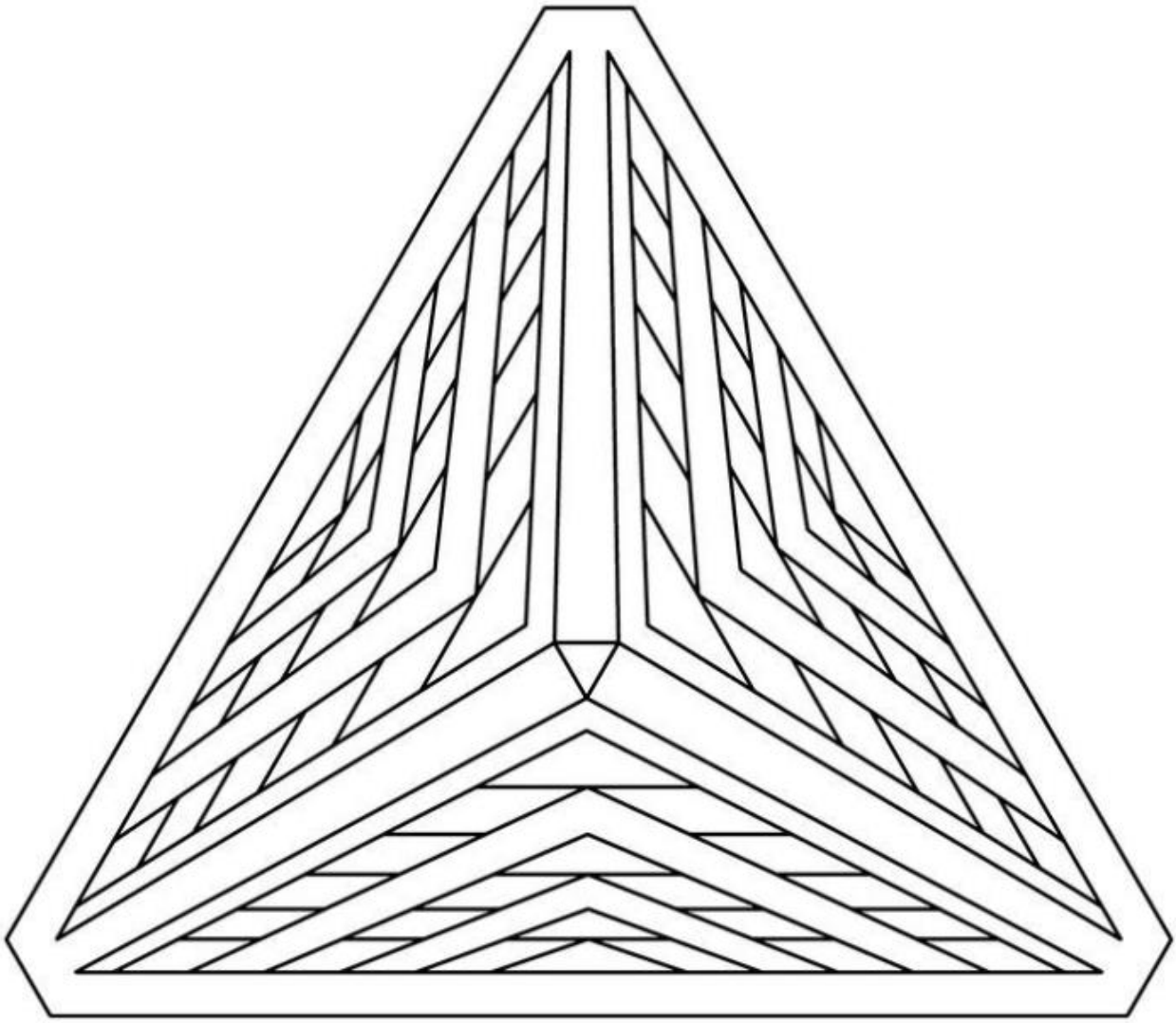

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